ELECTROMAGNETIC MODELLING OF RADAR SEA SPIKES

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INTRODUCTION

In this paper we describe the progress made in a project that seeks to characterise, first step towards this objective has been to establish and verify a capability to model sea clutter returns and their effects on the operation of radar systems in a way that is both realistic and makes direct contact with the prevailing environmental conditions.

Attempts to do this that have met with success in the past have been statistical in character and have always captured this environmental dependence through some essentially ad hoc parameterisation. So, for example, the K distribution model represents sea clutter returns as Gaussian speckle process whose local power itself is statistically varying and is represented quite adequately by a gamma distributed random variable [1]. The time and length scales characterising the correlation properties of these processes are widely different; this separation implicit in the compound form of the K process makes it particularly well suited to the analysis of the impact of clutter on radar signal processing [2,3]. Nonetheless the salient features of this model, the mean clutter power, its measure of spikiness or non-Gaussian character and its correlation properties, are effectively fed into such an analysis ‘by hand’.

The dependencies of some of these features on, for example, radar polarisation, swell direction and grazing angle, can be obtained from empirical formulae; these cannot be used with confidence outside the ranges of parameters for which they were originally established. Attempts to exploit simple models of both the sea surface and the understand and reduce the false alarm rate in maritime radars arising from the impact on their signal processing of spike like returns in the sea clutter. Our scattering process in the characterisation of sea clutter have not been successful. For example use of a Gaussian model of the surface height and a Kirchoff approximation to the scattering was not able to capture the essential features of sea surface returns at low grazing angles [4]. Analyses based on perturbation and composite model treatments of the scattering were similarly fruitless.

The failure of these models prompted more careful examinations of the low grazing angle scattering from specific, and realistically represented, sea surface features; these studies highlighted the role of breaking wave structures in the generation of sea spike returns [5]. Thus we identify the critical components of the required physically based model of sea spikes as the detailed modelling of radar scattering by breaking wave surface features and a statistical characterisation of the occurrence of these features in the ocean.

Significant progress has been made in the development of this model. We have developed a method for calculating low grazing angle (LGA) back-scatter from a rough imperfectly conducting surface that significantly extends and improves on methods described in the literature; this provides us with the first component of the model.

A large number of hydrodynamic models have been proposed that attempt to characterise the stability of the ocean surface; from these a consensus emerges that allows us to establish a simple criterion for the occurrence of breaking waves. This criterion, taken in conjunction with a statistical description of the sea surface in terms of a power spectrum and surface wave dispersion relation, provides us with our second component. Its analysis and implementation have required us to draw on, and in some cases, extend, on a significant body of oceanographic and statistical work.

A crucial test of our model is its ability to reproduce the dependence of mean radar cross section (RCS) on sea-state; we recall that existing simple models are not able to do this. Our model contains no free parameters beyond the input of environmental conditions through a model of the sea surface wave spectrum; we find that it produces very satisfactory agreement with observed experimental results.

ELECTROMAGNETIC SCATTERING

Over the past decade and a half the numerical calculation of LGA scattering by rough surfaces has been investigated quite intensively, motivated in part by its application to radar and other remote sensing applications. During that time the forward-backward (F/B) method of solution of the surface field integral equations (FIE) has emerged as a valuable technique for computing the scattering from perfectly conducting (PC) surfaces. This method was introduced originally by Holliday and co-workers [6]; several refinements have been developed by TWR [7] and have led to an efficient and robust implementation of a method whose underlying iterative method of solution has a transparent physical interpretation.

When the PC F/B method was applied to LGA scattering by realistic sea surface wave profiles it was found that several salient features of the observed scattering were absent from the output of the calculations. This deficiency could be rectified, if the imperfect conductivity of the sea was included; this provided a model that made direct and satisfactory contact with radar measurements made under controlled conditions in a wave tank, and was validated by numerical checks of reciprocity and surface field boundary conditions. The original calculations [8] that established this agreement employed a much modified F/B method, that did not share the speed and physical transparency of the PC implementation. Obviously it would be advantageous if we could recover, even to a limited extent, these advantages of the PC code and, at the same time, generate results that can be validated by comparison with the full imperfect conductor F/B code.

To do this we have treated the imperfect conductivity of the sea through the introduction of an impedance boundary condition (IBC). This modifies the Magnetic Field Integral Equation (FIE) that forms the basis of the F/B analysis of both Hpol and Vpol scattering by a PC surface. The extra term introduce in the Vpol case is relatively well behaved and poses no additional problems, beyond those encountered in the PC case, when the integral equation is discretised and solved by iterative matrix inversion. In the Hpol case, however, the IBC introduces a singular term into the kernel of the FIE, that has to be handled with considerable care. It is possible to avoid this problem by re-formulating Hpol scattering in terms of an electric FIE; by doing this, however, one loses contact with the existing PC F/B analysis of the problem.

Consequently we have analysed the singularity arising in the Hpol MFIE in some detail, and have devised a method that allows us to regularise the calculation in a way that leads to finite and sensible results.

Some indication of the problems encountered can be seen when the Hpol MFIE is applied to scattering by a simple planar interface. The requirement that we recover a simple Fresnel reflection coefficient in this case guided us through an analysis that gave us considerable insight into the practical implementation of the F/B method in this case. Thus we were able to discretise the MFIE occurring in this case, avoiding problems of spurious divergence that have marred earlier published discussions of this problem [9].

One of the principal difficulties faced in the calculation of back-scatter by an extended surface is the elimination of edge effects introduced by the finite size of the scattering surface accommodated within the computation. There are two ways in which this can be achieved. In the work of Holliday et al. on both the perfect and imperfect conductors, adjunct planes are introduced, extending from the scattering surface to infinity. The contributions of these surfaces are calculated semi-analytically and incorporated as extra inhomogeneous terms in the integral equations. Subsequent work, which has also focused on the exploitation of the impedance boundary condition, adopts a rather different approach: while Holliday et al. assume the incident field to be a plane wave, these workers weight the incident beam so that it illuminates only a finite part of the scattering surface. While this introduction of a weighted beam does eliminate edge effects from the calculation it makes the definition and calculation of RCS rather problematic, and yields results that cannot be checked straightforwardly against the validated output of the full imperfect conductor F/B calculation.

Our introduction of the IBC leads to extra terms in the adjunct plane contributions, much as it did in the discretisation of the FIE. The methods developed by Holliday et al for the evaluation of the PC adjunct plane contributions consist of the numerical quadrature in real space of functions that are oscillatory and slowly decaying. In particular, completely different methods are used to evaluate the forward and back-scattered adjunct plane terms. When these methods were applied to the additional terms arising from the IBC they were found to give inaccurate and inconsistent results. These problems were overcome by developing a rather different approach to the adjunct plane scattering. The integrands, whose long range and oscillatory character were the source of the difficulties encountered, were represented formally as contour integrals, chosen so that the integration over the surface of the adjunct planes could be carried out analytically. The remaining integral in the complex plane could then be carried out numerically, along a contour chosen so that the integrand was localised and non-oscillatory. This method allowed us to evaluate the extra adjunct plane contributions rapidly and accurately.

These developments have allowed us to construct an imperfect conductor F/B code that resembled the earlier PC code in many respects. Here we illustrate the effects of using the impedance boundary condition (IBC) and the choice of adjunct planes (AP) or weighted beam (WB) illumination. The scattering surface is a Gaussian shaped bump, which appears smooth at the radiation wavelength of 2 cm. Thus the scattering is mostly in the forward direction. Illumination is from the right at 10° grazing angle. Figure 1 shows the surface and full computation region of 7m. Figure 2 is the computed surface field for Vpol using the IBC; the perturbation to the surface field continues well beyond the Gaussian bump, right up to the end of the computation region. Figure 3 is a comparison of the bistatic scattering cross section versus observation angle for the Holliday method [8] and our IBC-AP approach. Figure 4 is a similar comparison with the weighted beam (IBC-WB) approach (the weighting being applied over the same computation region). The IBC-AP method compares well with [8] at forward scatter angles. The IBC-WB method, however, does not capture the forward scatter correctly because of the weighting applied to the field incident on the flat plane in front of bump, which affects the level of multipath interference. In backscatter directions both the IBC-AP and IBC-WB methods overestimate the scattering cross section due to the discontinuity in the field at the far edge (ie at +3.5m). Similar loss of accuracy had been encountered in our earlier development of the PC F/B code [7]; it is expected that the methods we employed in that case to recover this loss can be applied in the IBC case as well.

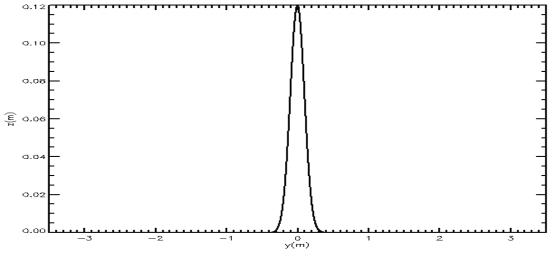

Figure 1: Scattering surface.

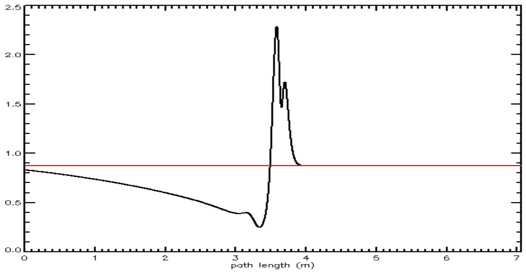

Figure 2: surface field for Vpol using IBC (flat plate in red).

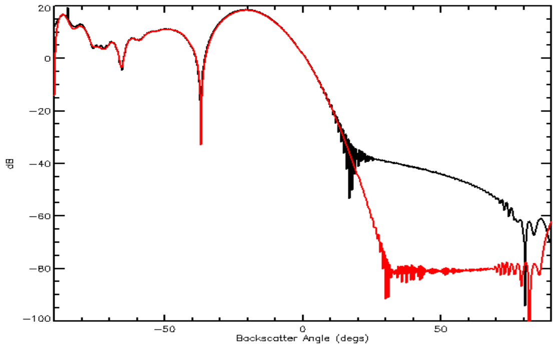

Figure 3: Bistatic RCS of [2] (red) and IBC-AP (black).

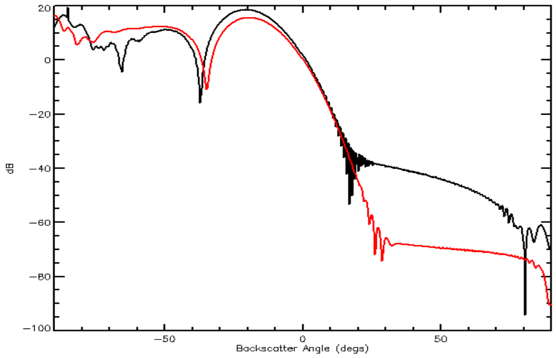

Figure 4: Bistatic RCS of IBC-AP (black) and IBC-WB (red)

STATISTICS OF BREAKING WAVES

The statistical characterisation of breaking wave events, whose scattering properties are to be calculated using the methods just discussed, provides the second component of our model. The changing shape of an individual breaking wave can be calculated directly, using methods described in the literature [10]. The statistical models that we use to predict and characterise the occurrence of these breaking wave events are not able to capture this level of detail. Thus the output of this specialised hydro code which must be introduced where necessary in our analysis. In particular, a Gaussian random field model of the sea surface cannot reproduce a breaking wave profile. Nonetheless it provides us with a direct route to a statistical model of sea surface properties such as its slope, velocity and acceleration, incorporating well established hydrodynamic models for the surface wave dispersion relation and power spectrum of the height fluctuations [11]. These make direct contact with the prevailing maritime environmental conditions and so have a predictive power that can take us into regimes where detailed clutter measurements have not been made. Several spectral models of this type are widely used in the oceanographic literature; [12] gives an extensive review of their physical basis and evidence adduced in their support.

To capture the simpler statistical properties of the breaking wave events, we use these models to ascertain the probability that threshold conditions for the occurrence of wave breaking are satisfied. Several such thresholds have been suggested in the oceanographic literature. We have adopted that based on the vertical acceleration of the surface [13] in part because of its relative tractability; there is also a body of evidence that there exists a significant measure of commonality between the various thresholds that have been proposed [14].

Once we have identified this threshold criterion, our analysis of the distribution of breaking waves reduces to that of above threshold excursions of a Gaussian random field with prescribed correlation properties. As such these are quite accessible to computer simulation; significant progress can also be made analytically in several areas. Thus the probability that a breaking wave occurs is given quite simply by the probability that the vertical acceleration (which is assumed to be a Gaussian random variable with a mean and variance that can be calculated from the sea surface power spectrum) exceeds a given threshold. This is no more than a simple 'probability of false alarm' problem. The properties of the local and global maxima and the supports of above threshold excursions required in the more detailed analysis of sea spike statistics are, in part, well documented in the literature [15, 16] while other topics are the subject of current research [17,18,19]. Taken together, however, this combination of analytical results and a readily realised simulation capability provide us with a useful toolkit for the characterisation of the occurrence of breaking waves and their contribution to radar sea clutter.

VARIATION OF RCS WITH SEA STATE

So far we have brought together and, where necessary, developed further, the EM scattering and statistical components of our model of maritime RCS. As we mentioned in our introduction relatively little progress has been made previously in establishing contact between even the simplest properties of low grazing angle sea clutter, such as mean RCS, and the prevailing environmental conditions. Thus a simple and yet stringent assessment of our model would be to test its ability to determine the variation of means RCS with sea-state and grazing angle. A well-established empirical model for the normalised RCS (NRCS) is the so-called GIT model [20]. Figure 5 shows the modelled variation of NRCS with grazing angle (0.1° to 10°) and sea-state (1 to 5), across wind at X-band with Vertical polarisation. Figure 6 shows the equivalent modelled variation with Horizontal polarisation.

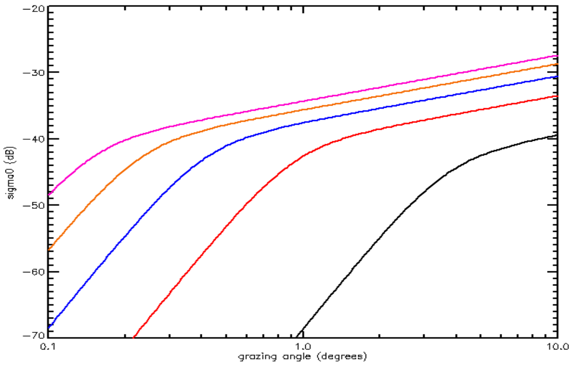

Figure 5: GIT NRCS model for Vpol. The black line is sea state 1, red 2, blue 3, orange 4 and purple 5.

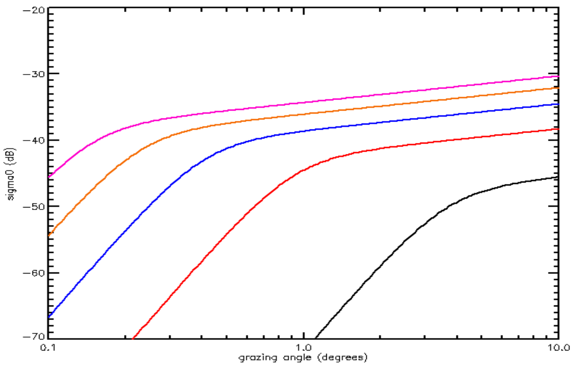

Figure 6: GIT NRCS model for Hpol.

For our calculations we use the Pierson-Moskowitz wave spectrum for a fully developed sea [21]. The composite model for scattering [22] is accurate at high grazing angles. It uses perturbation theory to derive scattering from small amplitude ‘resonant’ waves tilted by larger, longer waves. The overall RCS of the composite model is given by the average over the long wave slope distribution. Figure 7 shows the composite model applied to Pierson-Moskovitz wave spectrum. Comparing the results with the GIT model we see the composite model predicts the correct order of magnitude for V pol, but is much too low for H pol. Also, there is not sufficient sea state variation for both polarisations.

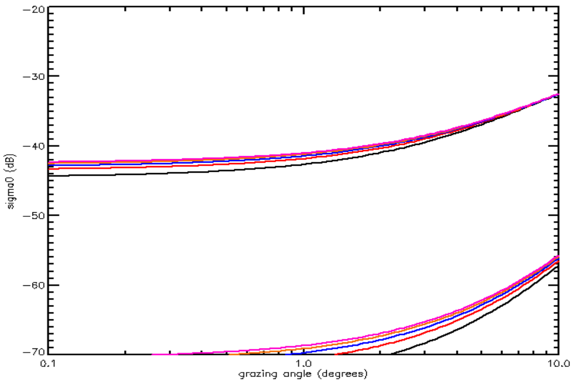

Figure 7: Composite model NRCS for Vpol (upper lines) and Hpol (lower lines). The colours relate to sea state, as in Fig5.

This later deficiency may be overcome by improving the model of the sea surface, and incorporating the change of wave spectrum in the gravity-capillary wavenumber regime, which was discovered and modelled by Jahne and Riemer [23]. With this modified wave spectrum the variation of NRCS with sea state is more realistic as shown in Figure 8.

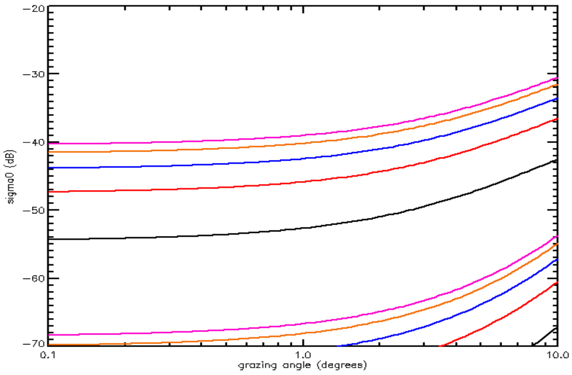

Figure 8: Composite model NRCS for Vpol (upper lines) and Hpol (lower lines) using the modified wave spectrum.

A further improvement to the composite model may be made through the introduction of shadowing and multipath interference. Shadowing reduces the amount of sea surface that is illuminated. At 10° grazing angle shadowing is negligible; below 10° we introduce an attenuation of 4 dB per decade. Multipath interference may be modelled on average using the roughness parameter

$$\rho = \frac{4\pi\sigma_h \sin\alpha}{\lambda}$$

where σ_h is the rms wave height, λ is the EM wave length and α is the grazing angle. The NRCS is then modified approximately by a factor

$$F = \frac{\rho^4}{1+\rho^4}.$$

The effect of the multipath factor F is to introduce a critical angle where $\rho = 1$. Above this angle the NRCS is largely unaffected, below it falls off by α^{-4}. With these additional factors the composite model NRCS is as shown in Figure 9.

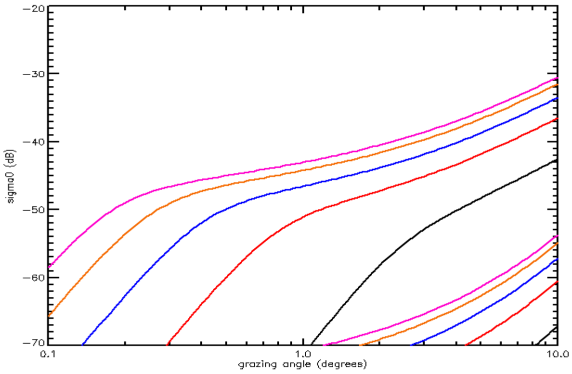

Figure 9: Composite model NRCS for Vpol (upper lines) and Hpol (lower lines) using the modified wave spectrum and including shadowing and multipath.

Results from scattering calculations show that the RCS of a breaking wave whitecap is largely independent of polarisation, and is approximately equal to the surface area covered by the whitecap. We can use the statistical model discussed in the previous section to evaluate the area covered by breaking waves; this is derived from the sea wave spectrum. Thus we can evaluate the mean NRCS of breaking waves and apply the shadowing and multipath factors discussed above. When the breaking wave results are added to the composite model Figures 10 and 11 are obtained. These graphs are remarkably similar to the GIT results plotted in Figures 5 and 6.

Furthermore, it is evident from the components of our model that V pol is dominated by composite model scattering, whilst H pol is dominated by breaking wave events. The intermittent nature of the latter provides a credible explanation for the more spiky nature of Hpol than Vpol.

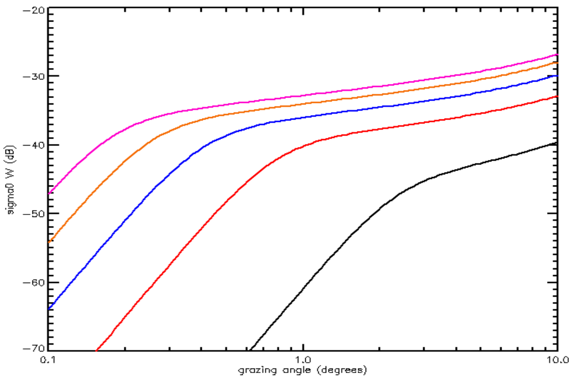

Figure 10: The sum of scattering from the composite model and breaking waves, giving the overall NRCS for Vpol.

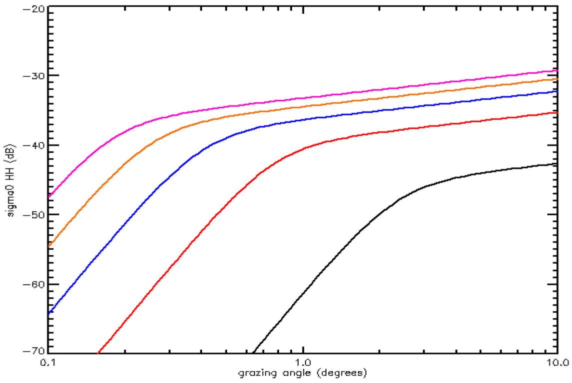

Figure 11: The sum of scattering from the composite model and breaking waves, giving the overall NRCS for Hpol.

DISCUSSION AND CONCLUSIONS

The model we have developed incorporates recent advances in the calculation of electromagnetic scattering by the sea surface, a mature and well-established understanding of the modelling of the sea surface, its dynamics and the generation of breaking waves and the statistical characterisation of above threshold excursions of random processes. Together these provide us with a means to relate the salient features of sea clutter directly to prevailing environmental conditions. Previous attempts to model sea surface scattering have failed to establish this connection, even in the simplest case of relating mean ocean RCS to sea-state.

In developing this model we have extended the F/B method to LGA scattering by an imperfect conductor, while at the same time retaining the relative simplicity and physical transparency of the earlier PC calculations. This we have done by exploiting the impedance boundary condition, and adapting, and so retaining, Holliday's adjunct plane construction to eliminate edge effects. The resulting code has been validated against the results of earlier, more computationally expensive methods.

A Gaussian model of the sea surface, with its structure and dynamics expressed through a power spectrum and dispersion relation, provides us with the starting point for our analysis of the statistics of the breaking wave events that contribute significantly to radar back scatter from the ocean. The contributions of these breaking waves to the radar back-scatter can be modelled using the output of the scattering code, at an appropriate level of detail. Thus in a calculation of the dependence of mean RCS on sea state, the EM scattering code provides an estimate of the RCS of a broken wave; in analysing the higher order statistics and their polarisation dependence, for instance, more detailed scattering calculations would be required.

Our model predicts a variation of RCS with sea state that reproduces quantitatively the behaviour of the GIT model, and yet introduces no free parameters (with the possible exception of the approximation for shadowing): our calculated RCS is a direct consequence of the input prevailing environmental and operational conditions. Thus we have calculated one of the two parameters characteristic of the K distribution, essentially from first principles. This preliminary success encourages us to extend our modelling to the higher order non-Gaussian statistics of sea clutter, and to their associated anomalously high and persistent returns. These in turn will furnish us with insights into the degradation of maritime radar performance, and suggest ways in which it might be retrieved through appropriate signal processing.

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