Multimode Local Oscillator Concepts for Improved Signal-to-Noise in Coherent Laser Radar

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Abstract

We describe the concept of multimode local oscillator field analysis for coherent laser radar systems and present new theoretical calculations of the improvements in heterodyne efficiency which can be obtained by these methods.

Keywords: Heterodyne detection. Coherent laser radar. Multimode waveguides.

Introduction

Coherent laser radar (lidar) systems are based on mixing the field received from a laser illuminated target with a local oscillator field. The resulting interference process provides significant advantages in sensitivity compared with direct detection techniques. The nature of the interference process is such that only transverse fields of identical form (i.e. non-orthogonal fields) contribute to the interference signal. Conventionally, the local oscillator takes the form of a TEM_{00} Gaussian beam from a laser source. In relation to the detection of speckle fields returned from laser illuminated targets, this has the disadvantage that only the TEM_{00} mode component of the speckle field is detected. This limitation is at the root of a number of shortcomings in the performance of lidar systems, including: (i) reduced detection sensitivity: all higher order spatial mode content remains undetected, (ii) inability to scale the receiver aperture, and hence, the signal-to-noise: conventionally the aperture diameter is chosen to match the diameter of a lobe in the speckle field, (iii) significant signal fluctuation: as a single speckle lobe moves across the receiver aperture the fundamental mode content shows large variations.

In earlier work, the possibility of making more comprehensive measurements of the field returned from a laser illuminated target by using a local oscillator which produces a range of orthogonal transverse modes was proposed and demonstrated [1,2]. As illustrated in figure 1, in this earlier work, a hollow multimode waveguide based version of the Michelson interferometer was used to provide the source for target illumination and to mix the field returned from the target with the output from the local oscillator, which could be one of a set of modes of the hollow multimode waveguide.

This approach should result in significantly enhanced signal levels and reduced signal fluctuation. The improvements in performance should be applicable across the electromagnetic spectrum to a wide range of coherent lidar applications. In this paper we report some theoretical calculations that have been carried out as a part of initial work on the current project. The aim of this work is to make a quantitative assessment of the value of applying this approach to measurements of a scattering target.

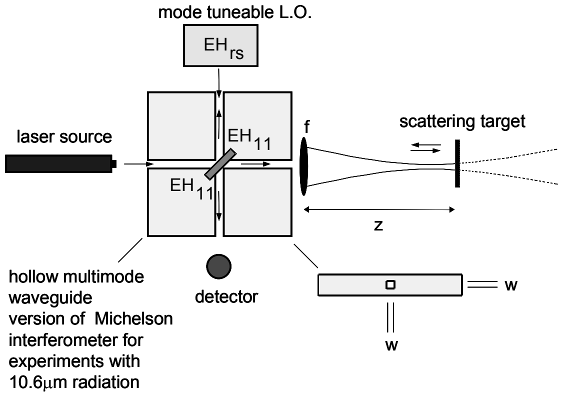

Figure 1: Schematic of multimode mode local oscillator field analysis concept, illustrating the use of a multimode waveguide version of the Michelson interferometer in the measurement of returns from a scattering target.

Theoretical Analysis

Our theoretical analysis is based on the setup shown in figure 1. This mono-static configuration is similar to that of many real Lidar systems, and forms the basis for our initial laboratory experiments. The fundamental mode output from the hollow multimode waveguide interferometer is focused by a lens of focal length f on to a scattering target at a distance z. Light which is back-scattered from the surface of the target returns through the lens and is coupled into the hollow waveguide interferometer. Here it is mixed with a local oscillator field which takes the form of a mode of the multimode waveguide. A critical factor determining the mode content of the speckle field is the characteristic speckle size of the scattered field at the point at which it enters the waveguide. The speckle size is maximised when the target is at the beam waist, which, for sufficiently strong focusing, will be in the focal plane of the lens. The speckle size reduces when the target is moved away from the waist. Thus, translation of the target allows us to investigate the effect of varying mode content.

A square cross-sectioned waveguide of width w supports a set of complete orthogonal modes (EH modes) with transverse fields given by [3]

$$E_{pq}(x,y)=\frac{2}{w}\sin\left(p\pi\left[\frac{x}{w}-\frac{1}{2}\right]\right)\sin\left(q\pi\left[\frac{y}{w}-\frac{1}{2}\right]\right) \quad .....(1)$$

where the x and y directions are parallel to the two sides of the waveguide, with x=0, y=0 being at the centre of the guide. The z direction lies along the guide axis. The mode numbers p and q are positive and start from 1.

The heterodyne signal produced from a photo-detector of area equal to or greater than that of the waveguide is proportional to an overlap integral:

$$i_{pq} \propto \int_{-\frac{w}{2}}^{\frac{w}{2}}\int_{-\frac{w}{2}}^{\frac{w}{2}} E_{pq}^{*}(x,y)E_0(x,y)\,dxdy \qquad (2)$$

where E_0 is the received field and a pqth mode is used as local oscillator. The modulus squared of the complex quantity i_{pq} is proportional to the fraction of the optical power in the received beam that has been converted to an electrical signal, which depends on how well the speckle field matches the local oscillator field. The heterodyne power will change if the back-scattered speckle field changes, due to relative motion between the beam and the scattering surface, for example. If there exists a well-defined statistical model for the speckle field, an average heterodyne power can be calculated, the average being over all possible configurations of the back-scattered field. Normalising this quantity by the average back-scattered power entering the waveguide, and by the local oscillator power, gives an averaged heterodyne efficiency

$$\eta_{pq} = \frac{\int_A\int_A \langle E_0(x,y)E_0^*(x',y')\rangle E_{pq}^*(x,y)E_{pq}(x',y')dxdydx'dy'}{\left(\int_A |E_{pq}(x,y)|^2 dxdy\right)\left(\int_A \langle |E_0(x,y)|^2\rangle dxdy\right)} \quad (3)$$

Here the angled brackets denote averages, i.e. expectation values, and the subscript A on the integrals denotes integration over the area of the waveguide. Since the waveguide modes form a complete orthogonal set, the sum of efficiencies over all values of p and q is equal to unity. The heterodyne efficiency thus tells us what fraction of the total available power is, on average, captured by any local oscillator mode.

Equation (3) can be evaluated by using the "back-propagated local oscillator method," [4]. It can be shown that the overlap integrals in (2) can be evaluated in any plane perpendicular to the propagation axis, with the local oscillator taken to be back-propagated out of the waveguide to that plane [3]. This allows one to evaluate the integrals in the plane of the scatterer, where, for a surface that is rough compared to the wavelength λ, the scattered field is delta-correlated [5].

$$\langle E_0(x,y)E_0^*(x',y')\rangle = \frac{\lambda^2}{\pi} I_0(x,y)\delta(x-x')\delta(y-y') \cdots (4)$$

The field at a distance z from the lens can be written as a Huygens-Fresnel integral:

$$E_{pq}(x,y,z) = \frac{ik}{w\pi z}\int_{-\frac{w}{2}}^{\frac{w}{2}}\int_{-\frac{w}{2}}^{\frac{w}{2}} \sin\left(p\pi\left[\frac{x}{w}-\frac{1}{2}\right]\right)\sin\left(q\pi\left[\frac{y}{w}-\frac{1}{2}\right]\right) \quad (5)$$
$$\times \exp\left(ik\left[\frac{x_1^2+y_1^2}{2f} - \frac{(x_1-x)^2(y_1-y)^2}{2z}\right]\right)dx_1dy_1$$

Here $k=2\pi/\lambda$. The efficiency can be calculated by substituting (4) and (5) into (3). In the denominator of (3), the power in the local oscillator mode is unity by consequence of the definition (1), and it can be shown that the average back-scattered power entering the waveguide from a scattering surface at z is $w^2/\pi z^2$. When the scatterer is placed exactly in the focal plane ($z=f$) the propagation integrals become Fourier transforms and can be easily evaluated analytically. In this regime, with suitably strong focusing, the transverse coherence length of the speckle pattern back at the waveguide is maximised and this maximises the heterodyne efficiency when a 11 mode is used as local oscillator. This is also equivalent to a far-field regime where the target is a long way away from the measuring system. Results are plotted in figure 2.

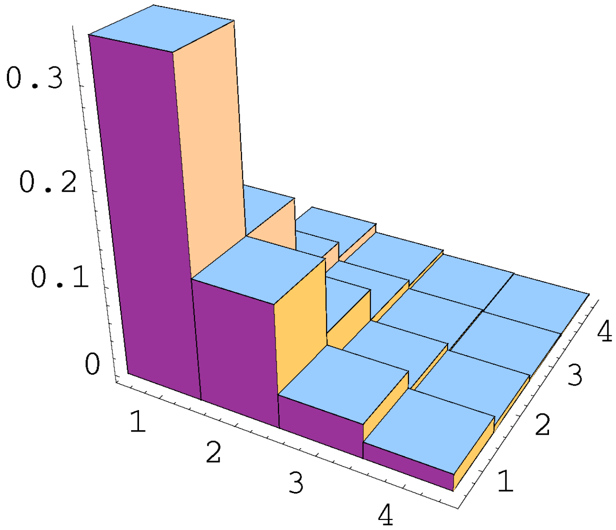

Figure 2: Heterodyne efficiencies in the focal plane (far field).

Note that there is complete symmetry between the pq and qp modes. As expected, the 11 mode has the greatest efficiency, 34%. There is also significant power available in some of the other low order modes: adding the 21, 12 and 22 modes gives a combined efficiency of 66%.

In the general case the integrals in (3) and (5) are less tractable, but can be evaluated numerically. It is convenient to use the dimension-less parameter $f' = 4f/kw^2$. Figure 3 gives an example of heterodyne efficiencies for $f' = 0.01$, which, for a 10.6μm laser, would correspond to a waveguide width of around 2cm (this could be system aperture which is imaged on to a smaller waveguide by relay optics) focused by a 50cm lens. In figure 3 z=0.93f, a value chosen to give a significantly, but not enormously, reduced value for η_{11}. At this distance the beam diameter is 50% larger than at the waist. Comparing figure 3 with figure 2, one can see that the 11 mode still has the greatest efficiency but that the proportion of power in the higher modes is greater. Therefore, there is more to be gained here by accessing higher order modes.

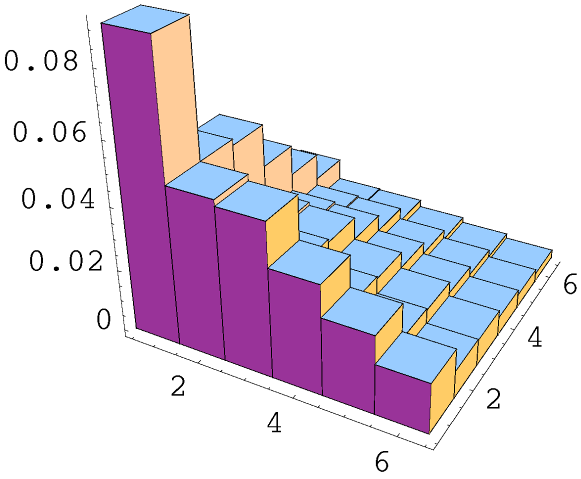

Figure 3: Heterodyne efficiencies for z=0.93f.

It is interesting to note that the best modes to use tend to be the *1q* and *p1* modes. Indeed, when the modes in figure 3 are listed in order of descending efficiency the first mode which doesn't have a 1 in its index, the *22* mode, is at position 8. Comparison with the earlier figure also indicates the greater potential for improving the cumulative efficiency in this regime: with the scatterer at the focus, the addition of 9 extra modes only increases the cumulative efficiency by just over 2 times, but the comparable result at z=0.93f is an increase of nearly five times.

Conclusions

The average heterodyne efficiency calculations presented in this paper quantify the amount of power available in different local oscillator modes and thus give an indication of the gains in performance that can be expected when simultaneously accessing a number of modes of the speckle field. It should be noted, however, that the improvement in overall system performance will depend on exactly what the system is measuring: the presence or absence of a target, a target Doppler shift or a vibration frequency, for example. Any calculation of these improvements would have to take into account the way in which the separate electrical signals are processed and combined, and the influence of measurement noise. These aspects will be addressed by future experiments and theory, which will also address issues associated with fluctuations in the scattered field. The development of the technology for simultaneous mode measurement will be a major part of the final two years of the project.

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