

## Netted radar and The Ambiguity Function.

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### Abstract

*In this paper, the key performance parameters of multistatic radar are investigated via the netted version of the ambiguity function. A new tool has been developed for assessing the resolution and ambiguity properties for network topologies consisting of  $N$  transmitters and a single common receiver operating in fully coherent or partially coherent modes. Simulation results outline the dependence of the multistatic ambiguity function on the relative positions of the nodes in the network and on the position of the target. More importantly it shows the potential for enhancements in resolution and for resolving ambiguities. Finally, the more complex case of  $N$  receivers and  $M$  transmitters is also considered.*

Keywords: Ambiguity function, netted radar, bistatic radar

### Introduction

Most current radar systems are monostatic i.e. the transmitter and receiver are co-located. The performance of this form of radar has been greatly enhanced by the advent of high resolution imaging, low side-lobe antennas, high speed digital signal processing and other technology improvements. However, it is well known that when a target is illuminated by electromagnetic radiation, scattering occurs in all directions. The receiver in monostatic radar will only intercept the very small portion of this energy radiated back in the direction from which illumination occurs and much of the signal and thus information is lost. Netted (or multistatic) topologies can help overcome this limitation and offer the potential to extend the capabilities and performance of current radar systems.

The implementation of such systems has now become feasible due to recent technology advances in other related fields such as communications (transmission lines with high capacity and wireless

technology), electronics (multi-channel antennas, electronic beam steering and high speed digital processors) and aerospace (precise synchronisation systems such as GPS), enabling low cost, coherent, secure and reliable radar networks [1].

Netted radar has some inherent advantages. For example spatial distribution of the nodes of the network enables surveillance to be tailored to the area of interest. In addition, it is possible to increase sensitivity, as more of the scattered energy (in the different directions) can be collected and hence detection performance can be improved. Target classification and recognition can also be enhanced, as the target is observed from different perspectives and hence more information is generated. Moreover, increased survivability and reliability is achieved because of the option of having 'silent' or passive operation of the receivers. These receivers can improve the location accuracy of possible jammers by fusing the information from the network nodes. Finally, if a single node of the network is

lost it can still provide a level of reduced performance and the network is said to exhibit graceful degradation [2].

### Background Theory

The ambiguity function is widely recognised as an important tool for evaluating the performance of monostatic radar. Woodward [3] has indicated its form:

$$\Theta(\tau, f) = |X(\tau, f)|^2 = \left| \int_{-\infty}^{\infty} u(t)u^*(t-\tau)e^{j2\pi ft} dt \right|^2$$

$\tau$  and  $f$  are the time and frequency shifts of the return signal.  $X(\tau, f)$  is the generalized auto-correlation function of any complex modulation  $u(t)$  and it can be regarded as the output of a matched filter receiver.

The resolution and the potential ambiguities of the system can be extracted by plotting the previous equation on the range – velocity plane. Such a plane is created by linearly transforming the original time delay – Doppler shift axis to the parameters of primary interest. An important observation for monostatic radar is that the resolution and the ambiguity performance are dependent only on the modulation and repetition frequency of the transmitted signal and not on the position of the target.

An alternative is a bistatic configuration where the transmitter and the receiver are spatially separated. Indeed this can be thought of as a component within a generalized netted system. I.e. the netted system is made up of a series of bistatic pairs. The ambiguity function for bistatic radar has been developed in [4] and is included here as a basis for describing ambiguity in netted radar. The ambiguity function for bistatic radar will have the same form as in monostatic radar, but the time and Doppler shift will depend on the baseline length  $L$ , the angle  $\theta_R$  of the receiver-target line and the angle  $\phi$  formed

by the velocity vector and the bistatic bisector [4], in a non-linear way. This results in the bistatic ambiguity function being dependent on the transmitter, target, and receiver geometry. A particular feature of the bistatic ambiguity function is the case where the target is close to the baseline. Here, the range and velocity resolution of the system is completely lost [4]. One method to avoid this is to employ a radar network.

### Netted Ambiguity Function

Here we consider netted scenarios. The topology selected is the simple case of  $N$  transmitters and one common receiver, i.e. we have in effect a series of multiple bistatic geometries with varying baselines. The reason for this choice lies on the fact that it is convenient to reference all calculations to the single receiver, thus obtaining one unified form for the ambiguity function.

The analysis is based on the matched filtering performed at the receiver. Before proceeding to the mathematical background, it is necessary to state the assumptions made when modelling the system. These are:

- i. The target is considered to be a slowly fluctuating scatterer.
- ii. The transmitted signal, after reflection by the target, is multiplied by the factor  $b$  which corresponds to the scattering characteristics of the target in the direction of the receiver.  $b$  is assumed to be a Gaussian random variable when the number of scatterers is large and none is dominant.
- iii. The target's scattering properties do not change with the angle of view.

It should be noted that the first two assumptions were developed theoretically in [4].

The transmitted modulation is common and has the following form:

$$s(t) = \text{Re}[u(t) \exp(j\omega_c t)]$$

$u(t)$  is the complex envelope of the signal and  $\omega_c$  is the carrier frequency. The return signal is an addition of the  $N$  scattered signals from the target due to each transmitter. A very important assumption made at this point is that coherent processing of the raw data is feasible. This implies that the  $N$  echoes, which arrive at different time instances as the transmitter – target – receiver paths are different, can be processed in phase. This will involve storing a number of returns, aligning them and feeding them to the matched filter. The received signal is:

$$r(t) = \sum_{i=1}^N \text{Re}\{b_i u(t - \tau_i) e^{j\omega_c(t - \tau_i)}\}$$

$\tau_i$  is the time delay,  $b_i$  is the multiplication factor. Taking into account assumption (iii),  $b_i = b = \text{constant}$ .

It must be noted that the echoes that arrive at the receiver do not have the same intensity, as the propagation lengths of the waves are different. Thus, a weighting must be applied, according to the signal power. The weighting factors are calculated by the following set of equations:

$$w_i = \frac{P_{Ri}}{\max(P_{Ri})}, \quad i = 1, 2, \dots, N$$

$$P_{Ri} = \frac{P_{Ti} G_{Ti} G_R \lambda^2 \sigma_B}{(4\pi)^3 (R_R R_{Ti})^2}, \quad i = 1, 2, \dots, N$$

The bistatic radar cross-section  $\sigma_B$  is taken to be constant. In the receiver, the weighted echoes are passed through a filter matched to the original transmitted signal. Following similar analysis to [4], and excluding  $b$ , the specific instance of the ambiguity function is given by:

$$X_{netted} = \left| \sum_{i=1}^N w_i X_i \right|^2$$

$X_i$  are the bistatic ambiguity functions for the different bistatic pairs.

In the case of  $M$  receivers and one common transmitter there is a significant practical limitation when attempting to implement this methodology. This is because it was assumed in our analysis that the scattered signals must be processed jointly. In the case of many receivers their inputs must be processed after correction for time discrepancies. An alternative approach is to process each echo in its own receiver before being transmitted to a central combining unit.

The outcome of this will be the same as in the previous case in terms of the system's parameters (resolution and ambiguity). However, the ambiguity function seen by each receiver will be individually different.

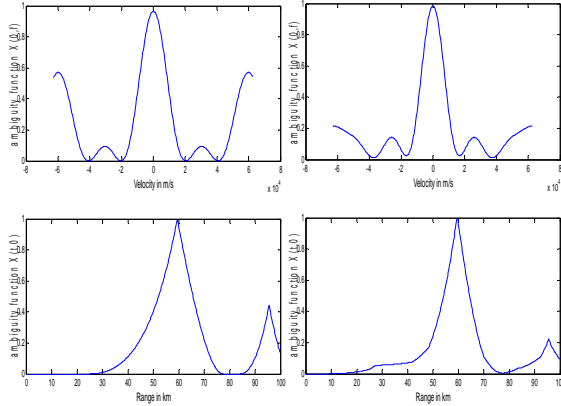
The mathematical representation of the ambiguity function for this second method is thus different. Matched filtering is performed for each echo and the final output is a summation of the bistatic ambiguity functions of the various bistatic pairs. The following equation outlines this:

$$X_{netted} = \sum_{i=1}^M W_i |X_i|^2$$

Examining the general case where a number of nodes are spatially distributed, a combination of the two previous methods can be used. That is each of the  $M$  receivers in the network will accept all the scattered signals, originating from the  $N$  transmitting stations, creating a series of multistatic ambiguity functions. These will be then used as inputs in the last equation to construct the radar equation ambiguity



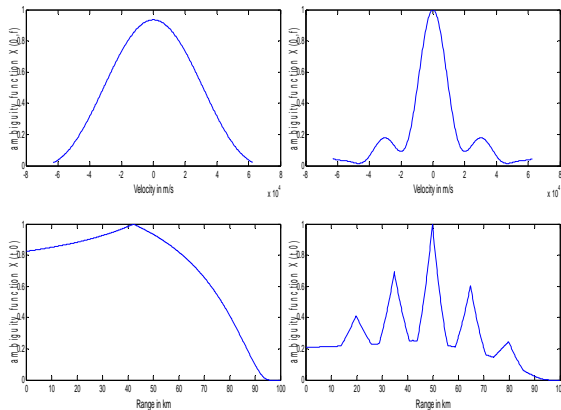
We now consider an alternative topology where the second transmitter is 100km above the receiver i.e. orthogonal to the bistatic baseline of the first pair. The angle  $\theta_R$  of the target is  $-60^\circ$ .



**Figure 4: Bistatic and netted cuts of the ambiguity function – balanced case**

The left plot of figure 4 represents cuts of the bistatic ambiguity function of the first bistatic pair i.e. in the absence of the second transmitter. Adding the second transmitter significantly improves the resolution in range and velocity. Moreover, the ambiguity peaks are suppressed.

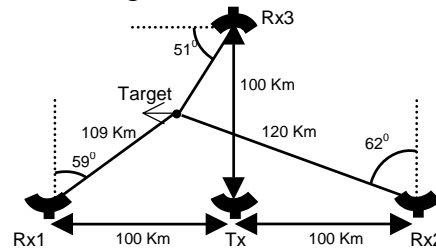
Returning to the geometry of the first example, where  $\theta_R = -80^\circ$ , an additional transmitter is placed in the position of the receiver, thus resulting in a network that is a combination of monostatic and bistatic radar. The results are shown in figure 5.



**Figure 5: Bistatic and netted cuts of the ambiguity function – varying the baselines**

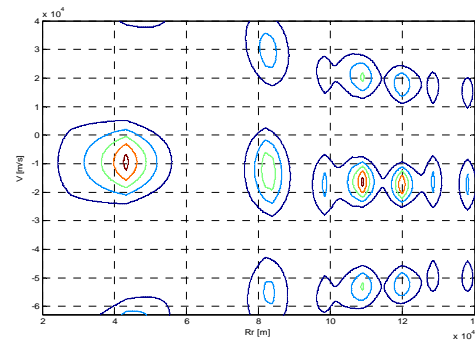
The improvement caused by exploiting spatial diversity is evident. In the simple bistatic case when  $\theta_R = -80^\circ$  the performance of the system is below the acceptable level (left plot of figure 5). Adding the second transmitter an appropriate position the resolution and ambiguity performance of the network allows us to extract some information about the target (right plot).

The final example involves a scenario where a number of receivers and one common emitter comprise the radar network. The specific geometry used is given in the figure 6:



**Figure 6: Geometry used**

The ambiguity diagram for this case is given in figure 7.



**Figure 7: Ambiguity diagram for a 3 receivers common transmitter scenario**

The resulting ambiguity is the summation of the three independent bistatic ambiguity functions in the range-velocity plane with appropriate weights. In the figure shown three main peaks can be identified. These correspond to the resolution as seen by the bistatic radars.

The differences between the two approaches ( $N$  transmitters/common receiver and  $M$  receivers/common emitter) for the netted version of the ambiguity function are clearer now. In the first approach a unified ambiguity diagram is obtained, where the overall properties of the netted system can be estimated. This includes the resolution capabilities and the ambiguity performance and is thus preferable. The second concept is to qualitatively assess the performance of the radar network. It is a visualization tool that allows the user to observe how the different bistatic pairs behave according to the relative positions of the nodes and the target. Thus a specific bistatic radar can be used to resolve ambiguities, whereas another one can provide the system with adequate resolution in the range and velocity domains. The different bistatic ambiguity functions co-exist in the generalized ambiguity diagram. In this way we can use the analysis to explore performance and optimisation of the behaviour of a distributed radar sensor for a given scenario and application.

### Conclusions and Future Work

The above netted scenarios indicate that detailed study must be performed for each particular topology in order to assess the capabilities of the radar network to locate a target to a known accuracy. Of particular interest is the case where the target is close to the baseline of the original bistatic pair and there is no resolution in range or Doppler. Adding more transmitters in the network at appropriate positions enables this undesirable property to be overcome thus enhancing the performance of the system significantly. The spatially separated transmitters can also be regarded as a very sparse array with associated ring-lobes appearing in the Doppler-range plane as ambiguities. This approach enables the use of techniques such as adaptive nulling to suppress ambiguity peaks through

appropriate spatial configuration of the nodes. It should be noted that this also highlights the fact that the system is likely to be severely spatially under sampled and care will need to be taken to control the magnitude and position of the inevitable ambiguities that will occur.

The work described here can also be extended to simulate scenarios with  $M$  transmitters and  $N$  receivers. The mathematical analysis presented allows this to be achieved and the next step is to obtain the results.

### References

- 1 Chernyak, VS , 1998, Fundamentals of Multisite Radar Systems, Gordon and Breach Science Publishers
- 2 Baker, CJ and Hume, AL, 2003, Aerospace and Electronic Systems Magazine, vol 18, issue 2, 3-6
- 3 Woodward, PM, 1953, Probability and Information Theory, Pergamon, New York
- 4 Tsao, T, Slamani, M, Varshney, P, Weiner, D and Schwarzlander, H, 1997, IEEE Trans. in Aerospace and Electronic Systems, vol 33, No 3, 1041-1051

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