Present Status of the Compact EM THz Source

R.A. Stuart, C.C. Wright, A.I. Al-Shamma'a
Liverpool John Moores University,
General Engineering Research Institute,
Byrom Street, Liverpool L3 3AF

Abstract

In this paper, the overview of the current THz systems are reported and the small signal gain of a folded waveguide travelling wave tube (FWTWT) is calculated by means of Madey's theorem. The results of this analysis were compared with a single particle simulation carried out with MATLAB. The results were in excellent agreement. The calculations indicate the great potential of this device as an oscillator in the terahertz regime.

Keywords: THz, FWTWT, FEL, waveguide, gain

Introduction

To develop an electromagnetic (EM) wave coherent radiation source in the THz gap, which as shown in figure 1 exists between microwaves and IR radiation in the EM spectrum i.e. between 100GHz and 10THz (λ = 3mm to 30µm).

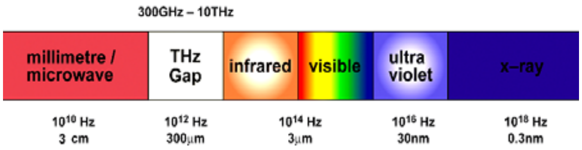

Figure 1 *Electromagnetic Spectrum*

There are currently only a few THz radiation sources available and these can only produce low output beam powers (of the order microwatts to milliwatts). These are summarised in figure 2 and listed in table 1 and based upon the current technologies of semiconductor electronics and optical devices.

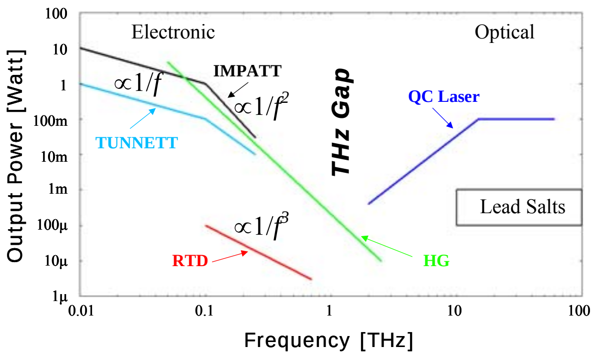

Figure 2 *Current technologies cover both electronic and optical devices*

The main devices listed in table 1 [1] indicates a lack of power sources (<<1W) over the entire frequency region 0.01THz to 100THz but also no substantial devices exist within the THz frequency gap ranging from 0.1THz to 10THz. These are the two main features, which this THz project planning to research.

Sources (Instrument)	Frequency (THz)	Power
Si – Impatt devices	0.4THz	2mW at 77°K
RTD resonant tunnelling diodes	0.7THz	1µW
GaAs Tunnett diodes	0.2THz (2nd Harmonic)	10mW at 300°K
Schottky Multipliers (mm – wave sources)	0.4THz, 0.8THz, 1.2THz	12mW, 2mW, 0.2mW
Heated powder films	60THz	100µW
Quantum Cascade Laser	2THz to 60THz	0.1mW to 100mW

Table1 *Current THz specifications*

Overview

The modelling and simulation of the FWTWT have been carried out to quantify the THz source out put characteristics (gain and power). The structure of a FWTWT is shown in figure 3.

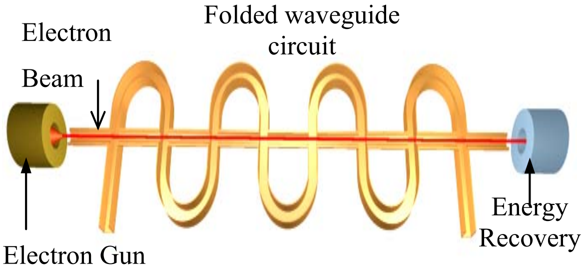

***Figure 3** FWTWT THz System*

A rectangular waveguide, in which the dominant TE_{10} mode is propagating, formed into a serpentine shape by a series of E-plane bends [2]. Axial holes in the structure allow an electron beam to cross the narrow dimension of the waveguide many times. In each gap, the electric field is in the same direction as the electric field of the electromagnetic wave, so that if the gap is sufficiently narrow, considerable energy can be exchanged with the electrons in the beam.

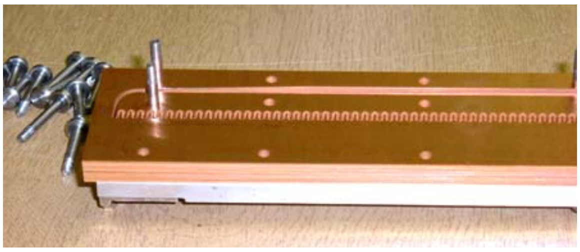

***Figure 4** Stack of laminations mounted onto side plate*

Figure 4, shows the complete stack of copper laminations made by Photofabrication process and placed on one of the two 6mm thick aluminium side plates. The THz system setup is shown in figure 5.

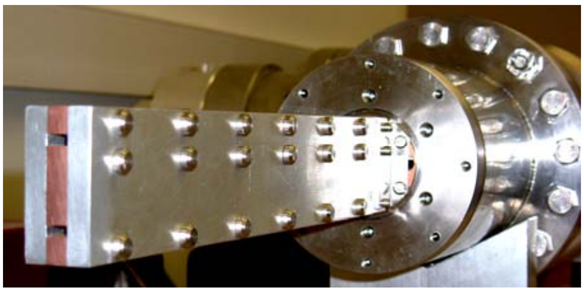

***Figure 5** Assembled FWTWT system*

The waveguide structure of the FWTWT is made by assembling a set of copper laminations containing a serpentine cut out, input and output waveguide tapers and an electron transport channel. Other potential THz sources were investigated including the electrostatic system. The electrostatic device works in a very similar fashion of the free electron laser devices, but now the static voltages on the electrodes imposes a spatial modulation of the electron velocity. Technically, spatial harmonics, which imposed on the electron beam, rather than on the electromagnetic wave as in the case of the FWTWT, in order to achieve a gain condition. The results obtained from such a system have proven that although the mechanical simplicity of the system, but it has a very low percentage gain <<1% if operating at voltages <10kV, and 10% gain under electron beam operation of >400kV with an out power in the region of micro watts.

Theoretical analysis

The plan layout of a FWTWT illustrating all the dimensions used in the analysis is shown in Figure 6. The narrow face of the rectangular guide forming the folded waveguide is of width b, while the broad face is of width **a**, (into the paper). A TE_{10} electromagnetic (e-m) mode is assumed to be propagating in the +z'-direction, and an electron beam is passed through the device along the z-axis from left to right.

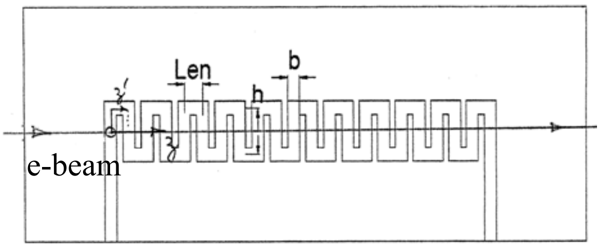

***Figure 6** A plan view of a folded waveguide travelling wave tube, showing the height of each fold h, and the half pitch, len. The rectangular waveguide narrow side dimension is b, and the wide side dimension, a (into the paper.*

It has been shown [3] that the interaction between the electron beam and the e-m mode in the guide can be thought of in terms of the interaction of electrons with a fictitious continuous slow forward wave that propagates along the z-axis. The electric field component of this "spatial harmonic" is given by

$$\vec{e}_o^+ = A\frac{b}{len}snc\left(\frac{k_0^+ b}{2}\right)\cos(\omega t - k_0^+ z).\hat{z} \qquad (1)$$

where $$k_o^+ = \frac{\pi}{len} + k_g\left(\frac{len+h}{len}\right) \qquad (2)$$

and A is the amplitude of electric field of the TE_{10} mode in the guide having frequency, ω, and wavenumber, k_g. This z-directed field will affect the energy of the electrons such that

$$\frac{d}{dt}\left(\gamma mc^2\right) = -e\cdot\vec{e}_0^+ \bullet \vec{v} \qquad (3)$$

The left hand side of this equation represents the rate of increase of the energy of an electron, while the right hand side is the rate at which work is done on the electron by the electric field. We approximate the right hand side by assuming that the electron velocity is hardly changed by the interaction, so that $\vec{v} = \vec{v}_0 = v_{z0}\hat{z}$ where v_{z0} is the unperturbed electron velocity. This implies that the amplitude, A, of the e-m mode is also hardly affected so that it may be taken as a constant. Consequently, we can write equation (3) as ,

$$\frac{d\gamma}{dt} = -\frac{e}{mc^2}A\frac{b}{len}v_{z0}snc\left(\frac{k_0^+ b}{2}\right)\cos(\omega t - k_0^+ z + \phi) \qquad (4)$$

where a phase, φ, has been introduced to allow for a range of entry times of an electron into the system. By dividing both sides by v_{z0} and noting that, in this approximation, $dz = v_{z0}dt$, we get

$$\frac{d\gamma}{dz} = -D\cdot\cos(\omega t - k_0^+ z + \phi) \cong -D\cdot\cos(\Delta k.z + \phi) \qquad (5)$$

where $D = \frac{e}{mc^2}A\frac{b}{len}snc\left(\frac{k_0^+ b}{2}\right)$ and $\Delta k = \frac{\omega}{v_{z0}} - k_0^+$

Integrating both sides of equation (5) from z = 0, to z = N.len (i.e. over the length of an N-fold FWTWT) gives the overall *change*, γ_1, in relative electron energy from entrance to exit as

$$\gamma_1 = -\frac{D}{\Delta k}\sin\left(\Delta k.z + \phi\right)\Big]_0^{N.len} \qquad (6)$$
$$= -\frac{2D}{\Delta k}\sin\left(\frac{\Delta k.N.len}{2}\right)\cos\left(\frac{\Delta k.N.len}{2} + \phi\right)$$

If the electron entry times are imagined to be distributed uniformly in phase from -π to +π, then equation (6) shows that as many electrons will gain energy as will lose energy, so that no net exchange of energy between the electrons and the e-m wave appears to be possible. We express this by stating that the average energy change per electron is zero or

$$\langle\gamma_1\rangle = 0 \qquad (7)$$

where <···> indicates averaging over the phase, φ.

However, as noted earlier, several approximations have been made in deriving equation (8), so that this is only a first order result. (Hence the suffix 1 in γ_1). If better approximations are made, it is found that there is a small but finite change in total energy of the electrons, so that to second order, we have

$$\langle\gamma_2\rangle \neq 0 \qquad (8)$$

The importance of Madey's theorem [4,5] lies in the fact that it is possible to relate the wanted, but normally difficult to calculate second order energy change, $\langle\gamma_2\rangle$, to the previously (and easily) calculated, first order energy change, γ_1. Madey's theorem states that

$$\langle\gamma_2\rangle = \frac{1}{2}\frac{d}{d\gamma}\langle\gamma_1^2\rangle \qquad (9)$$

so that if we find the average of the *square* of γ_1, we can find $\langle\gamma_2\rangle$ directly by performing the required differentiation.

Squaring Eq. 6 gives,

$$\gamma_1^2 = \left(\frac{2D}{\Delta k}\right)^2\sin^2\left(\frac{\Delta k.N.len}{2}\right)\cdot\cos^2\left(\frac{\Delta k.N.len}{2} + \phi\right) \qquad (10)$$

which on averaging over φ gives

$$\langle\gamma_1^2\rangle = \frac{1}{2}\left(\frac{2D}{\Delta k}\right)^2\sin^2\left(\frac{\Delta k.N.len}{2}\right) \qquad (11)$$

since $\cos^2(\varphi + \mathrm{const})$ when averaged over φ is equal to 1/2. Substituting this result into Madey's theorem, we find that

$$\langle\gamma_2\rangle = \frac{1}{4}(D.N.len)^2 \frac{d}{dx} snc^2\left(\frac{\Delta k.N.len}{2}\right)\Bigg]_{x=\frac{\Delta k.N.len}{2}} \frac{d}{d\gamma}\left(\frac{\Delta k.N.len}{2}\right) \quad (12)$$

with snc(x) = sin(x)/x. Now

$$\frac{d}{d\gamma}\left(\frac{\Delta k.N.len}{2}\right) = \frac{N.len}{2}\frac{d}{d\gamma}\left(\frac{\omega}{v_{z0}} - k_0^+\right) = \frac{N.len.\omega}{2}\frac{d}{d\gamma}\left(\frac{1}{v_{z0}}\right)$$

$$= -\frac{N.len.\omega}{2.v_{z0}^2}\frac{dv_{z0}}{d\gamma} = -\frac{N.len.\omega}{2.c.\beta_{z0}^3\gamma^3} \quad (13)$$

where $\beta_{z0} = v_{z0}/c$. If the electron beam current is I_e, then I_e/e electrons pass into the device per second so the total power *lost* by the electrons (note the change of sign) and therefore the power *gained* by the e-m mode is equal to

$$P_{out} - P_{in} = \langle\gamma_2\rangle mc^2 \frac{I_e}{e} \quad (14)$$

$$= \frac{1}{4}(D.N.len)^2 \frac{d}{dx} snc^2\left(\frac{\Delta k.N.len}{2}\right)\Bigg]_{x=\frac{\Delta k.N.len}{2}} \frac{N.len.\omega}{2.c.\beta_{z0}^3\gamma^3}\frac{mc^2}{e} I_e$$

where P_{in} is the e-m power entering the device and P_{out} , the power leaving.

The gain of an FEL [4] is defined as the ratio of this *increase* in power gained by the e-m mode to the input e-m power, i.e. $(P_{out} - P_{in})/P_{in}$. (Note this differs from the gain usually employed by electrical engineers, which is the ratio of power out to power in, P_{out}/P_{in}). For a TE_{10} mode in a rectangular guide the input power is given by

$$P_{in} = \frac{1}{4}\frac{A^2}{120\pi} ab \frac{ck_g}{\omega} \quad (15)$$

so the *FEL gain* is given by, (16)

$$G_{FEL} = \frac{\left(\frac{e.N.b}{2mc^2}\right)^2 snc^2\left(\frac{k_0^+ b}{2}\right)\cdot\left(\frac{N.len.\omega}{2.\beta_{z0}^3\gamma^3 c}\right)\cdot\left(\frac{mc^2}{e}\right)\cdot I_e \cdot \frac{d}{dx} snc^2(x)}{\frac{1}{4}\frac{ab}{120\pi}\frac{ck_g}{\omega}}$$

where the differential coefficient is evaluated at x = Δk.N.len/2. The electronic gain, G_E is given by $G_{FEL}+1$. Eq. (16) could be simplified by cancelling terms but there is little justification for this since the whole expression is straightforward to programme via MathCAD. The results of running this software for a FWTWT design loosely based on that given by Booske [7] for operation at 560GHz are shown in Figure 7. The FWTWT dimensions were as follows: a = 300μm, b = 43μm, len = 66μm,h = 70μm

(Note these dimensions are not all the same as quoted by [7], since the use of semicircular folds in the design, not the sharply bent folds analysed here.

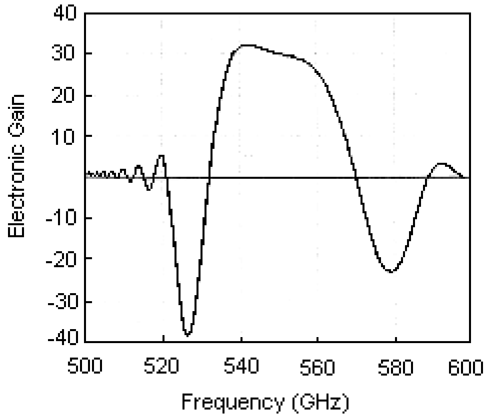

***Figure 7** The variation of the electronic gain with frequency*

The device was assumed to be driven with an electron beam accelerated to 10.64keV at a current level of 0.5mA. The number of folds, N, was taken as 255, giving an overall length as 16.83 mm. The calculations show that the maximum *electronic* gain was 32X, or 15.1dB at a frequency of 542 GHz. Booske's simulations were performed with state of the art TWT software, and gave a gain of about 13dB at the same frequency. However, Booske's device apparently consisted of just 100 folds compared with the 220 used in our simulation. Since the gain predicted is proportional to N^3, the agreement therefore appears much better than it actually is. The possible reason for this discrepancy is that the approach adopted here using Madey's theorem is a *small gain* model. This means that for the theory to be applicable, the amplitude, A, of the e-m mode should be hardly altered by the interaction – i.e. the model can only accurately predict small gains. As can be seen from Figure 7, the calculated maximum gain is very large, and in no way could the output power be said to be only fractionally larger than the input. This inconsistency is emphasised by the fact that the electronic gain, P_{out}/P_{in}, cannot be less than zero *by definition*, since P_{out} must be greater than or equal to zero. However, Figure7 shows that regions of frequency with gain less than zero are in fact predicted. In practice, the model can only be used with any certainty provided the electronic gain calculated is limited to less than about 2. However, it appears that in a FWTWT the interaction is so much stronger, that the small gain approach can only be taken to give a ball park figure of the gain if the results give values much greater than 2.

A further objection to the theory is that it is a *single electron* model that is electron – electron interactions are neglected. These are better known in the electron device world as space charge effects. They are well known to have a serious influence on the gain of electron beam/e-m wave devices such as TWTs and klystrons. Pierce's theory of the TWT [8], which does include the effects of space charge, shows that after the e-m wave has travelled a short incubation distance through the device where its growth is low, the gain of the wave becomes *exponential*, i.e. much more rapid than the N^3 dependence predicted by the Madey's theorem approach. It may well be that our theory applies only to this initial, low gain region, i.e. for short length and/or low current devices. These points illustrate the fact that a fuller theory of the FWTWT using a Pierce type approach (or equivalently a coupled mode approach) is an important area for future work.

Finally, it could be that some numerical oversight or error has been made in deriving our expression for the gain. To check this, a numerical small gain simulation has been carried out.

Numerical simulation

The basic equations controlling the motion of electrons in a FWTWT are given below (Eqs. 17 to 19). These equations are based on the following assumptions:-

(1) The electron beam current is sufficiently low that space charge effects can be neglected, i.e. this is a single particle model.
(2) The interaction is sufficiently weak that the amplitude and phase of the e-m mode can be regarded as independent of position in the FWTWT, i.e. this is a small gain model.

These assumptions are the same as those made earlier for the Madey's theorem treatment but now we can remove the assumption that the electron velocity is constant.

Conservation of energy and Newton's force law then give

$$\frac{d\gamma(t)}{dt} = -\frac{e}{mc^2} A \frac{b}{len} v_z(t) snc\left(\frac{k_0^+ b}{2}\right)\cos(\omega t - k_0^+ z(t) + \phi) \quad (17)$$

$$\frac{d\left(\gamma(t) m v_z(t)\right)}{dt} = -e \cdot A \frac{b}{len} snc\left(\frac{k_0^+ b}{2}\right)\cos(\omega t - k_0^+ z(t) + \phi) \quad (18)$$

with $\frac{dz(t)}{dt} = v_z(t)$ (19)

These equations have to be numerically integrated over the length of the device. This operation is simplified if differentials with respect to distance rather than time are used, by replacing dt by $dz/v_z(t)$.

A MATLAB programme has been written to solve these equations. A set of electrons uniformly distributed in phase from $-\pi$ to $+\pi$, are simulated passing through the device, and their final energies, $\gamma(N.len).mc^2$ are averaged. Any net decrease in energy is equated to a net power increase of the e-m mode, and hence the gain can be calculated.

Figure 8 shows the results of a simulation. The number of folds and the operating parameters of the simulated FWTWT were chosen so that small gain conditions applied. The calculated gains at spot frequencies were compared with the output of the Madey's theorem programme. The agreement was within 2%, even when only 10 electrons were used in the simulation. Even better agreement could be obtained when more electrons were used, but at the expense of running time.

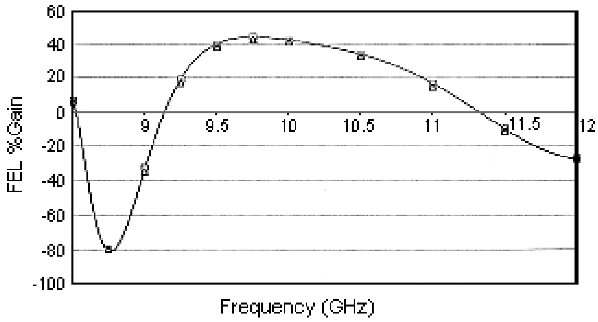

Figure 8 *Gain vs. frequency for a nominal 10GHz FWTWT*

Perhaps the agreement obtained is not that surprising since both Madey's theorem and the simulation assume the interaction is between an electron and a continuous *spatial harmonic* wave passing along the axis of the device. In reality, the interaction is between an electron and a spatially discontinuous electric field that only exists in the waveguide gaps of the FWTWT. To check this, we have programmed another simulation, this time using expressions for the *exact field* in the gaps. The gain results are included in Figure 8. Again the agreement with the other methods is excellent in spite of the fact that only 10 electrons were used. The run time of this programme was however much longer than the spatial harmonic one.

Summary

The gain of a FWTWT in the small gain, single electron regime has been investigated. A simple analytical formula derived by using Madey's theorem has been developed. The results of this analysis have been confirmed numerically by two simulation procedures, one involving representing the interacting electric field in the device in terms of a spatial harmonic, the other in terms of the exact field in the gap experienced by the beam electrons. Although the gain formula can only represent a practical situation provided the maximum electronic gain is less than about 2, the formula does provide a quick, ballpark figure for the gain of any particular device that is useful in the initial stages of a design. Furthermore, it has been suggested that the major use of these devices would be as a source in the terahertz region. From our experience with FEL oscillators, it may be a positive advantage to operate the FWTWT in the small gain regime where Madey's

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