Optical Aperture Synthesis

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Abstract

Analysis of images recovered from 'snapshot' synthesis instruments are used to assess the power requirements for active imaging systems. For extended targets the pulse powers required and the experimental design appear feasible for operation in the thermal infra-red and marginal for application in the near IR.

Keywords: Synthesis imaging, active imaging

Introduction

In previous meetings in this series we have described [1, 2] the principles of synthesis imaging and, in particular, its potential use in active imaging for military surveillance. In particular, we have considered the process of Fourier Telescopy as a mechanism by means of which one may mitigate the deleterious effects of atmospheric turbulence in high-resolution imaging in the wavelength range from 10μm to the visible region. In this paper attention is directed more towards an analysis of system-type issues in order to understand the practical hurdles that will need to be surmounted if practical implementations of synthesis imaging in military applications are to be feasible.

Signal to noise considerations

The signal to noise (SNR) for a Fourier Telescopy measurement follows essentially the same analysis as is appropriate to conventional optical aperture synthesis. In essence the process is one in which the dominant noise *in the measurement* is presumed to arise from photon-counting statistics [3]. This formulation remains true for various signal (flux) levels from high levels down to single-photon counting observations – although the dominant terms in the analysis change dependent on the signal strength. Notwithstanding the laser pulse powers that may be indicated, the received signal in an active surveillance application will be low and, certainly when evaluating performance limitations, the low-flux limit is the appropriate regime to consider. Here we will be considering the phase associated with a photon-limited signal and, as a multi-valued quantity, the use of the phase variance is a better measure of performance than an SNR measure in this modulo arithmetic. The phase variance on such a measurement may be expressed [3]

$$\sigma_{\varphi}^{2}(\xi) = \frac{1}{2n\left|J(\xi)\right|^{2}} + \ldots, \qquad (1)$$

where n is the mean photon flux received in a snapshot measurement and $J(\xi)$ is the normalised Fourier transform of the signal. For a synthesis imaging observation, expression (1) must be recalling that for a given flux level and for a system composed of an array of N identically-sized apertures equation (1) may be replaced with

$$\sigma_{\varphi}^{2}(\xi) \sim \frac{N}{2m\left|O(\xi)\right|^{2}} + \ldots, \qquad (2)$$

where now m is the detected photon flux per aperture and $O(\xi)$ the Fourier transform of the target reflectivity. If we assume a boring target (i.e. a disc that is p pixels diameter in a diffraction-limited system with a pupil diameter equivalent to the largest array dimension) then the appropriate subsidiary peaks in $|O(\xi)|^2$ fall off roughly as the inverse of the number of pixels squared, for images of the complexity that we need to consider here a disc 14 pixels across will give $|O(\xi)|^2 \sim 10^{-4}$ and a disc 39 pixels across will give $|O(\xi)|^2 \sim 10^{-5}$. These values refer to the local maxima in the function and it will be fortuitous indeed if the synthesis array sampled at the peaks, however targets showing structure will scatter more strongly at large ξ-values and the values given above may reasonably be taken for use in performance estimates.

Image quality

For a filled-aperture optical system the Strehl ratio, S, is often used as measure of image quality. For a system with small optical defects the Strehl ratio can be related to σ_φ^2, the phase variance (due to optical imperfections), by the expression

$$S \sim \exp\left(-\sigma_\varphi^2\right). \tag{3}$$

Physically the Strehl ratio is the ratio of the axial height of the system point-spread function (psf) in the imperfect system compared to the height of the psf peak in a diffraction-limited version of the system. Whilst there appears to be no *a priori* reason for the same relationship to hold in a synthesis system (especially when the number of apertures is small) simulated results show that the same expression is a good approximation to the ratio of the psf peak height in a diffraction-limited synthesis array to the psf peak in an aberrated array.

For a filled-aperture system achievement of a Strehl ratio above about 0.3 means that the visual image looks good, below this value the observer is aware of the image defects. How does this relate to image quality in a synthesis system where the diffraction-limited image will still suffer from side-lobe induced defects?

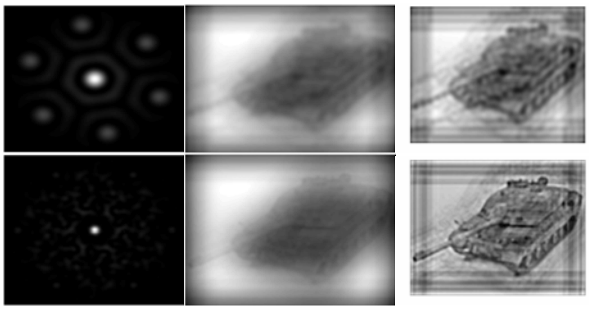

Fig 1: Left: psfs from arrays with 9 (top) and 21 apertures. Centre: the raw tank images. Right those images re-processed. These represent a ‘no noise’ ideal.

The ‘reprocessing’ shown in figure 1 consists of simply re-weighting the calibration baselines in the synthesis arrays and can be achieved in real-time during image synthesis. The ‘ringing’ at the edges of the square frame is an artefact that would be absent when soft-edged illumination is used. However, such ‘images’ may display unphysical negative intensities and will not be considered further – instead we concentrate on raw images of the type shown in the centre in figure 1.

The synthesis arrays considered here are all consistent with the use of Redundant Spacings Calibration [4] (RSC) and in assessing likely instrument performance the residual errors after calibration need to be taken into account. We have therefore conducted a series of simulations in which 100 images of the target were assessed at each of several different phase variance values. At phase variance values of 0.25, 0.5, 0.75 the Strehl ratios expected on the basis of expression (3) are 0.78, 0.61 and

0.47. To assess ‘image quality’ a series of images were synthesised. For different levels of post-correction phase variance the Strehl in each ‘snapshot’ was evaluated and the probability density function (pdf) for the Strehl plotted, along with the cumulative distribution function (cdf).

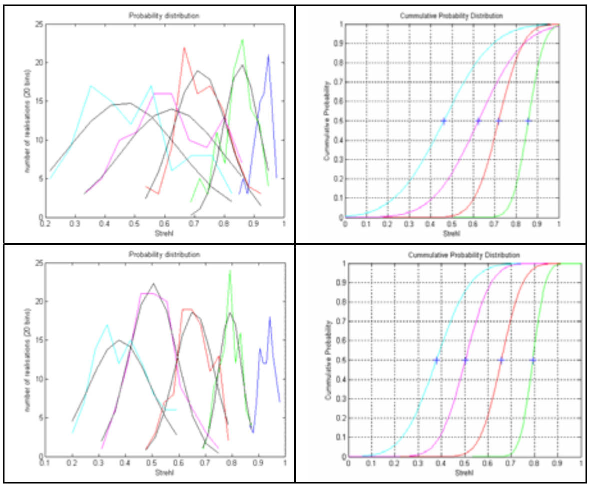

Figure 2: pdf and cdf for the Strehl ratio from 100 simulations at each phase variance. Phase variances are: 1 (cyan); 0.75 (magenta); 0.5 (red); 0.22 (green) and 0.1 (blue) – all values in rad^2. Top row is for a 9-aperture system, bottom row for a 21-aperture system.

The black curves on the left hand side in figure 2 represent Gaussian fits to the pdfs and from these the cdfs shown on the right have been obtained.

On the basis of heuristic assessments of the image quality we adopted a requirement for performance that there should be a 95% probability that a ‘snapshot’ Strehl is $\geq$ 0.3. This criterion gives a sufficiently-low probability that two successive ‘snapshots’ fail to give an image suitable for target recognition. From the cdf, this leads us to require a phase variance $\leq$ 0.75 rad^2.

Lucky synthesis imaging

Before proceeding we note that the use of a synthesis technique implies a small number of degrees of freedom in the system and, thus that the probability that a set of random and independent aberrations co-align to give a low aberration (thus, high-quality image) is usefully different from zero. This is effectively the ‘luck observer’ model used foe a long time in solar astronomy and recently exploited with great success in night-time observations [5].

The ‘lucky’ probability can be deduced from the cdf, which implies that a residual variance of 0.75 rad^2 will give a better than 50% chance that a ‘snapshot’ delivers a ‘good’ image (better than 0.5 Strehl).

Laser power requirements

Combining equations (2) and (3) gives

$$m \sim -\frac{N}{2\ln(S)\left|O(\xi)\right|^2}. \qquad (4)$$

For a target that acts as a Lambertian scatterer with albedo a, is at range R and where the scattered radiation is collected through apertures of diameter d and detected with overall optical efficiency q, we have

$$p \sim -\frac{4hcNR^2}{q\lambda a d^2 \ln(S)\left|O(\xi)\right|^2}\,\text{J}, \qquad (5)$$

As the requirement for the laser pulse power at wavelength λ, h, c being Planck’s constant and the speed of light respectively.

However, we should note that the Strehl ratio in expression (5) was expressed in terms of the *residual* phase error in the synthesis array. In order for the *residual* error to be below a given level one must allow for the inevitable amplification of the raw-data error during data inversion (i.e. array calibration) however this is achieved (thus, even if iterative rather than analytic data inversion is used).

The condition number, κ, associated with the inversion of the RSC algorithm provides an upper bound on the error amplification and can be included in expression (5) to accommodate the error amplification due to data inversion, giving

$$p \sim -\frac{4hcNR^2}{q\lambda a d^2 \kappa \ln(S)\left|O(\xi)\right|^2} \text{ J.} \qquad (6)$$

However, κ is an upper bound on the error amplification and a more pragmatic approach is to compute the covariance of the error on the raw data and allow for the error amplification in terms of β the mean value on the diagonal of $\left(A^T A\right)^{-1}$, where A is a sparse matrix required to solve the RSC calibration problem [6].

This process has been carried out on a number of RSC array designs, and leads to the following comparison for arrays:

Array #	κ	β
1	20.7	9.2
2	34.1	25.5
3	48.4	16.1
4	72.4	17.2
5	18.9	8.4
6	90.2	29.4
7	18.7	8.3
8	37.8	5

Table 1: Comparison of condition number κ and a more realistic estimate of error propagation β.

As shown in table 1 the condition number is generally pessimistic by a factor of 2-3 but there are examples (note particularly array # 8) where the condition number is particularly pessimistic.

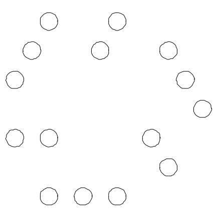

Figure 3: Layout for array #8 from table 1. This array has good conditioning and a near-circular shape gives ideal spatial frequency coverage.

In what follows, therefore, an error amplification by a factor of 10 will be taken as a representative number for κ in equation (6).

Image information content

Thus far we have avoided discussion of the image information content required for recognition tasks. This subject is best treated by reference to the standard approach enshrined in the Johnson criterion [7]. These suggest that 5-8 line pairs across the target are required if identification of that target is required. Thus, identification requires that the synthesised image has a minimum format of 14x14 pixels in order to allow for 7x7 line pairs on the target.

Conveniently, a 14x14 image contains 196 pieces of information. An N-aperture synthesis array contains $N(N-1)/2 = 105$ pieces of independent complex data for a 15-aperture array, thus 210 independent pieces of information. Since RSC requires $N-2$ pieces of information the 15-aperture array maps well onto the Johnson criterion.

However, Fourier telescopy [2] operates through the projection of cosine^2 fringes onto the target, whereas in order to evaluate the target Fourier transform one would, ideally, project cosine fringes. The cosine^2 fringes can, however, be regarded as a double-frequency cosine superposed on a uniform background. But in this case some extra degrees of freedom must be used to evaluate the level of this background (in principle this level is known but in practice there will be stray flux and de-coherence effects to be calibrated through this route).

Further, real images are contaminated by noise and many other sources of error, plus the fact the synthesised resolution does not map perfectly to an arbitrary target. Adding to this the fact that poor signal to noise can lead to a requirement to increase the Johnson criterion by a factor of 2-3x, it seems likely that synthesis arrays of 18, or even 21, apertures are likely to be required.

These factors may be combined in order to provide an estimate of the laser pulse power required for synthesis target recognition. Assuming a target albedo of 0.1, an array of apertures of 10cm diameter, a target power spectrum of 10^{-4}, an overall optical efficiency of 0.1, and a required Strehl ratio of 0.5 one obtains:

Wavelength	*N*	Pulse energy
1.5μm	9	0.41J
10μm	9	0.06J
1.5μm	15	0.52J
10μm	15	0.09J
1.5μm	21	1.9J
10μm	21	0.3J

Table 2: Laser energy required in the imaging pulse after allowing for specific array design, Johnson criterion, calibration errors and Strehl ratio requirements.

The figures in table 2 refer to specific array designs, in the case of the 15-aperture system the array modelled is shown in figure 3 and in the case of the 9 and 21-aperture systems the arrays used correspond to those modelled when computing the data presented in figure 1.

Other than when using a CO_2 laser source these laser powers appear to be higher than one would have wished.

Discussion

Synthesis imaging using Fourier telescopy and active imaging appears to be an attractive proposition for operation in the thermal infra red, where suitable cw sources are available, but the estimated pulse energy looks too high for operation at shorter wavelengths.

There are other issues associated with the use of this active-imaging technique that have not been discussed here. Principal amongst these is the question of the data diversity to be employed in the detection scheme. For each degree of information that is to be recorded in a single pulse one requires a data diversity scheme that encodes that information unambiguously.

Approaches to data diversity are temporal diversity (a comb of separate sort duration pulses delivered in raid succession within the relaxation time of the turbulence and/or platform vibration spectrum), frequency diversity (simultaneous use of closely similar frequencies where the frequency spectrum transmitted through different array pairs encodes the information in the beat frequencies produced) and polarization diversity.

Of these the latter appear unattractive both because it offers limited scope for diversity and because the targets are likely to be de-polarizing.

Examination of the number of degrees of freedom required in the measurement suggests that a combination of the other two approaches (especially using commercial WDM techniques) does offer a viable solution to this issue.

References

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