Re-Construction Algorithms for Aperture Coded Imaging Systems

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Abstract

Aperture coded imaging systems offer a number of potential benefits over conventional optical systems. These advantages include reduced cost and weight associated with the ability to use less expensive optical components. The resulting defects, such as poor depth of field, can be corrected in the digital domain once the intensity image has been collected. Other advantages can include robustness to countermeasures and in the case of adaptive systems, very rapid optical adjustment. In moving away from traditional optical components a trade-off has been made, where the advantages mentioned come at the expense of considerably increased computational requirements, whose monetary cost has steadily fallen. However, there is still a gulf separating the performance capabilities of current real-time image re-construction algorithms from more intensive non-real-time re-construction algorithms such as those used in astronomical applications. In many cases the real-time algorithms introduce some additional artefacts or enhance high spatial frequency noise. In this paper, a number of approaches are investigated based upon parallel techniques, simplifications of current non-real-time approaches, and new approximations designed to try to bridge this divide in a computationally efficient manner.

Keywords: Computational Imaging, Wavefront Encoding.

Introduction

Aperture encoding [1] uses a phase mask, placed at the aperture of an imaging sensor, to modify the optical transfer function (OTF). This introduces a known optical transformation and renders the resulting imagery less sensitive to de-focus and other optical problems. Digital correction for this known transform can simultaneously enhance the depth of field of the system and potentially also correct other optical defects. This paper discusses the development and investigation of methods for speeding up the processing of wavefront encoded imagery.

Aperture encoded systems allow numerous potential benefits to be exploited in the design of sensors and their optics, enabling depths of fields to be increased whilst minimizing the size and mass of the system. Performing real-time image reconstruction at 25Hz is possible by using a fixed Weiner filter [2]. However, this method, although optimal amongst linear operations, can lead to artefacts in the reconstructed imagery. Other better performing, non-linear techniques, which have shown great utility in non-real time applications can be used in an iterative manner, although these are significantly more computationally demanding.

The objective of this work was to attempt to bring some of the benefits of the more sophisticated non-real time approaches to bear on the real-time problem via novel parallelisation or approximation schemes. Two approaches have been devised that both have general applicability to a wide range of different digital reconstruction techniques. The first approach sub-divides an image into a set of sub-images and processes them independently whilst the second approach uses a novel pixel-based processing scheme that is rather like a Cellular Automata.

Digital Image Reconstruction Algorithms

The reconstruction of an image can be described by a linear model illustrated below in Figure 1. The overall goal is to recover image L from a given image I through knowledge of the optical transfer function F in the presence of noise N.

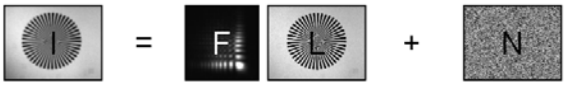

Figure 1 – Linear model of the reconstruction method

There are two approaches that can be used to reconstruct the image: firstly, the application of a fixed inverse filter; or alternatively, an adaptive filter which changes based upon image content.

A simple inverse filter can be readily applied to correct for a known fixed transformation of an image, e.g. the blurring caused by an imager's point spread function. However, when noise is present, a filter is required that can allow the source image to be reconstructed whilst minimizing the effects of noise. The direct application of the inverse filter F^{-1} leads to the amplification of the noise term. When the noise characteristics are known, the Weiner filter is the optimal fixed linear filter. Where all the noise characteristics are not known, the Weiner filter can be approximated by the inverse filter with an additional amount of low pass-filtering applied to prevent noise amplification. The extra filtering is computed from the noise power spectral density gained through working in the Fourier domain. The translation to and from the Fourier domain can be computationally expensive. However, recent advances in digital signal processing have enabled dedicated hardware to be developed that can implement the translation to and from the Fourier domain in real-time.

More advanced techniques apply a non-linear approach to re-constructing the image producing higher quality imagery. These techniques have been widely adopted by the astronomy community for high quality image correction and enhancement where processing time is not a major driver. There are two main approaches: the Maximum Entropy method [3], which adopts a forward modelling approach to directly find the un-degraded image within a certain noise tolerance governed by the level of noise; and the Lucy Richardson method [4,5], which also uses a forward modelling approach to derive an approximation to the maximum likelihood solution.

The Maximum Entropy approach seeks to arrive at the signal set that incorporates all known, testable information about the target scene. At the same time it is maximally non-committal about what is not known within the image.

The Lucy Richardson approach uses prior knowledge of the noise model to derive the maximum likelihood solution. Both methods require knowledge of the Point Spread Function (PSF) that has degraded the imagery. Through an iterative process, the re-constructed image is derived without the need to translate imagery into the Fourier domain. The Lucy-Richardson approach is computationally somewhat faster than the Maximum Entropy approach. The aim of this work was to arrive at a number of novel methods that would enable enhancements to be made in the speed of implementation whilst maintaining the quality of the resulting output. This was to be achieved through the consideration of parallelisation of the processing methodology and novel computational ideas.

Parallelisation Via Sub-Images

The sub-division of the image into a number of smaller sub-images should enable an image to be processed more quickly through the use of a parallel approach when compared to the time taken to process the image as a whole. Using this approach, the sub-images are processed independently of each other through the application of a digital image reconstruction technique.

The input image is divided into a multiple square sub-image blocks which include a processing border or 'guard region' which is sacrificed when reconstructing the final image. The guard region is used to reduce the effect of ringing in the processed sub-images. To this end, the original image is extended through the use of reflective boundary conditions at the image edges. The Lucy-Richardson de-convolution was applied to each sub-image block in turn using 'a priori' knowledge of the coding plate's PSF with the aim of reducing the effect of the noise in each sub-image block. The resulting reconstructed image is assembled from the core regions of each sub-image block.

The results of the localized approach were compared to the global application of the Lucy-Richardson de-convolution in order to assess its comparative performance. A number of image quality metrics were also used to characterize performance of the local and global techniques against the source image prior to the application of noise and blur artifacts.

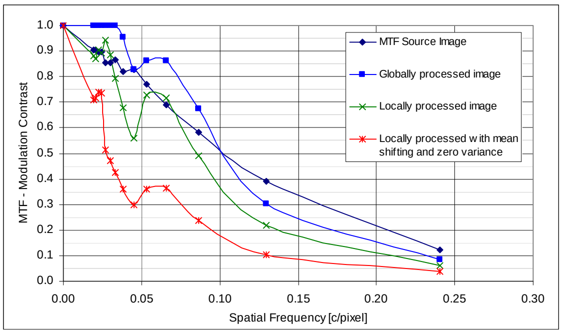

Figure 2 – Variation of Modulation Contrast with spatial frequency for globally and locally processed imagery

The MTF's for the different methods of processing the blurred and noisy images illustrate some of the artifacts that result from this processing methodology. The reduction in the MTF for the locally processed image with mean shifting between 0.025c/pixel to 0.05c/pixel is as a result of the change in the background luminance level in the central section of the test pattern. The origin of the MTF values higher than the source image arise from cases where the contrast between the bright and dark spokes of the target have maximum contrast. These mainly occur in the globally processed image. The limiting spatial frequency (equivalent to an MTF=0.05) for the locally processed images is around 0.25c/pixel. For the globally processed image this is closer to 0.26c/pixel whereas the source image has a limiting MTF of 0.28c/pixel.

Pixel Processing Approach

An even more extreme form of parallelisation can be envisaged where independent computations are carried out at the pixel level. This could potentially have a number of advantages for speed of operation provided that the necessary computations can be carried out by a sufficiently simple processing unit at the pixel level. In theory, such a structure could be implemented on a field programmable gate array (FPGA). The diagram in Figure 3 illustrates the concept. Here each pixel of the imager has a simple dedicated processor that can perform simple arithmetic operations, has a few registers to store intermediate values and a permanent memory for weighting coefficients or other program data. If possible, only nearest-neighbour pixel communications is desirable since any communication significantly greater than this is likely to become a processing bottleneck.

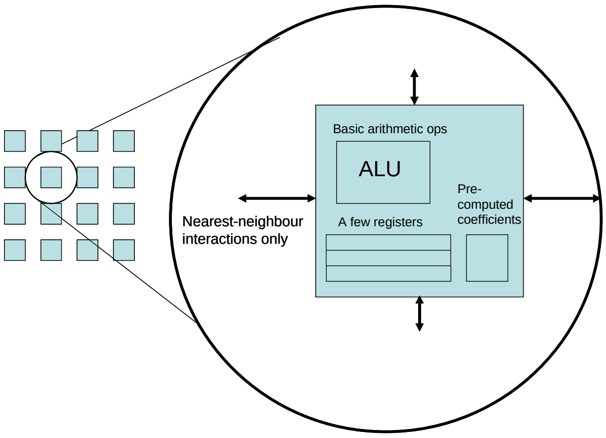

Figure 3 – Pixel Processing Concept

It is perhaps surprising that such a simple device, replicated for every pixel can in fact perform fairly sophisticated image processing operations. The number of cycles required per frame also need not be high, perhaps as low as a few thousand or even a few hundred. The next section illustrates some useful image processing algorithms that have been investigated within this architecture.

The method by which image processing, and in particular filtering operations, can be achieved within the architecture described above is to exploit the communications with neighbours. Thus a pixel could compute an average of its own value with those of its neighbours. Since all pixels perform the same function this is equivalent to the application of a simple blurring filter. Performing a second iteration of this process is equivalent to a two-fold application of this filter, or alternatively the application of a filter defined by the convolution of a neighbour averaging filter with itself. If an iteration-dependent weight factor is applied at the same time then a range of different filters can be obtained. In the image domain this can be represented by:

$$I_{out} = \sum_{n=0} w_n P^{\otimes n} I_{in}$$

where I_{out} is the output image, I_{in} is the input image, w_n is a weighting factor and $P^{\otimes n}$ represents the n-fold convolution of a pixel level filter with itself. In fact P can be any linear operation computable via only local pixel interactions. In the Fourier domain this relationship becomes:

$$f_{out} = \left(\sum_{n=0} w_n \tilde{P}^n \right) f_{in}$$

where f has been used to denote the Fourier transformed image. Now the operator is diagonal and therefore the iterated convolutions become simple powers. One can in fact regard the set of operators $\{1, \tilde{P}, \tilde{P}^2, ...\}$ as a basis for representing filters in the Fourier domain. This basis is not orthogonal (which is not very important), and is not complete, which means that not all filters can be represented in this way – some degree of approximation will be required. Given a filter that is required and an underlying local pixel-level operation (such as averaging neighbours), the weights required to best approximate it via iterated convolution can be computed using a variety of approximation methods. For this work a linear least-squares approximation based upon the Fourier domain approach was used.

If the filtering operation required is more complex, and in particular if it is asymmetrical, such as is the case for the PSFs involved in aperture coded imaging systems, then additional sophistication can be included within this process. The first step is to try to find the optimal local pixel operation as well as finding the optimal weights once this is known. This has also been implemented via a steepest-descent procedure. This is initialised with a simple averaging operator and the optimal weights are found via least-squares. Then a small perturbation is made to the averaging filter and a new set of optimal weights found. The mean-squared error of these approximations to the desired filter can then be computed. The difference between these is a numerical approximation to the derivative of the local pixel operator in the 'direction' of the perturbation. This is used as the basis for a steepest descent optimisation procedure that finds a good local pixel operation for the particular global filter required.

Results

These two approaches have been applied to both real and simulated data-sets of wave-front encoded imagery. A number of image quality metrics were used to assess the comparative performance of the two techniques, but for brevity in this paper just the Modulation Transfer Function (MTF) results are shown. It is interesting to see how well the pixel-based technique is able to represent the PSF of the coding plate even though it is severely asymmetrical. Figure 4 and Figure 5, show the true and approximated PSF and Figure 6 summarises the PSF results of the techniques described.

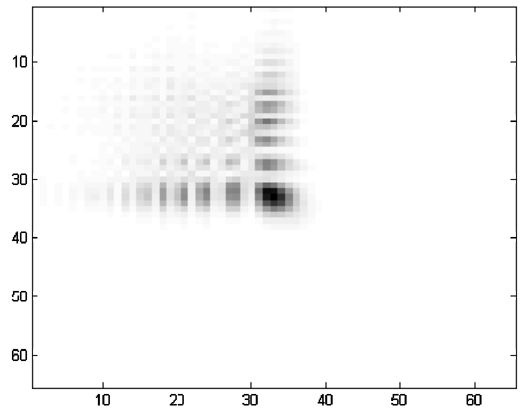

Figure 4 – Real PSF of coding aperture

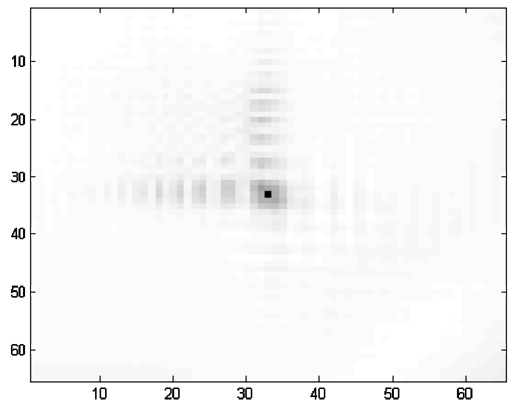

Figure 5 – Approximated PSF of coding aperture

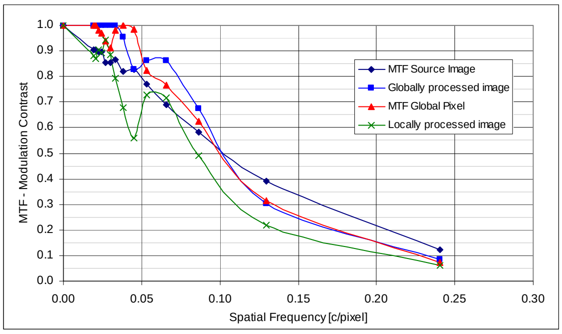

Figure 6 – The variation of Modulation Contrast with spatial frequency for a range of processing methodologies

The 'globally processed' image curve in Figure 6 is the baseline for comparison since this processes the entire image without regard for computational time or cost. Both the approximated approaches described (which parallelise the global Lucy-Richardson algorithm) are able to deliver performance that is close to the best possible.

Discussion and Conclusion

This paper has demonstrated the principles of implementing sub-image processing using a Lucy-Richardson de-convolution. Additionally, a novel pixel based method has been described which approximates the PSF applied to correct for the effects of a known encoded waveplate.

The sub-image based processing methodology, although not requiring fewer overall computations than the global method (due to the use of the guard regions and other technical restrictions that cannot be discussed in this paper due to lack of space) will allow performance enhancements to be made. These performance enhancements can be realised by dividing the image into a number of blocks which are then processed using a highly parallel processing architecture.

The pixel based approximation proposed is reliant upon using a discrete number of pixels in the immediate vicinity of the pixel of interest. This local pixel based processing architecture could be implemented within the focal plane array or using an FPGA. The benefit of using the pixel based approach is its simplicity with each pixel based processing block only required to undertake a few hundred operations per frame compared to the many thousands associated with the local area and global processing techniques described. Using this pixel based method, simple PSFs can be successfully approximated, but errors are introduced for more complex PSFs. Despite these errors the structural form of the reconstructed image is retained to a similar level of detail achieved by applying a globally processed Lucy-Richardson de-convolution.

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